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An alternative technique for the computation of the designator in the retinex theory of color vision

(Mach bands)

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ABSTRACT **Accepting the first postulate of the retinex theory of color vision that there are three independent lightness-determining mechanisms (one for long waves, one for middle waves, and one for short waves), each operative with less than a millisecond exposure and each served by its own retinal pigment, a basic task of retinex theory becomes the determination of the nature of these mechanisms. Earlier references proposed several workable algorithms. [Land, E. H. (1959) *Proc. Natl. Acad. Sci. USA* 45, 115–129; Land, E. H. (1959) *Proc. Natl. Acad. Sci. USA* 45, 636–644; Land, E. H. (1983) *Proc. Natl. Acad. Sci. USA* 80, 5163–5169; Land, E. H. & McCann, J. J. (1971) *J. Opt. Soc. Am.* 61, 1–11; Land, E. H. (1986) *Vision Res.* 26, 7–21.] The present paper describes a relatively simple alternative technique for the computation of the designator in retinex theory and reports the general operational effectiveness of the new technique, including the competence, not possessed by earlier algorithms, for generating Mach bands.**

On the basis of experiments described in previous papers (1–3, 5), we start from the first postulate of retinex theory: There are three independent lightness-determining mechanisms—one for long waves, one for middle waves, and one for short waves—each served by its own retinal pigment. A basic task of retinex theory becomes the determination of the nature of these mechanisms.

Earlier references proposed several useful algorithms (1–5). This paper will describe a new and relatively simple alternative technique for the computation of the designator in retinex theory. The designator is the computed numerical measure on one waveband of the lightness seen as part of the whole field of view. Previous retinex techniques have involved some kind of comparison between the flux (on one waveband) coming to the eye from a point on the object and flux (on that same waveband) arriving from points in remote, as well as contiguous, areas. These comparisons involve edges, gradients, thresholds, and pathways, and provide the average of the relationships between a given point and a large number of other points in the field of view. The criteria, as in all retinex theory, were that the value determined be independent of uniformity and intensity of illumination and be achievable with an exposure of <1 msec; i.e., independent of adaptation. Keeping the same criteria, the new technique, instead of utilizing an average of these relationships, compares the flux from the point of interest to an average, weighted in an unusual way, of the fluxes from all points in the field.

It is easily shown that this average cannot be a simple average taken over the whole field of view. Fig. 1 shows a collage of black, white, and grey areas, randomly distributed and randomly surrounded. If area 1 reflects 8% of the light falling on it, area 2 reflects 30%, and area 3 reflects 80%, the first will look dark, almost black, the second will be a middle grey, and the third will be almost white. If the illumination is so adjusted by neutral wedges in the illuminator that the flux to the eye from R_1 equals the flux to the eye from R_2 equals the flux to the eye from R_3, the observer will scarcely notice. The nearly black area stays dark, the middle grey area stays grey, and the white area stays white—even though the three measured fluxes to the eye are now identical. If we take the ratio of the flux from R_1 to the average flux for the whole field, and the ratio of the flux from R_2 to the average flux for the whole field, and the ratio of the flux from R_3 to the average flux for the whole field, these three ratios will equal each other. Therefore, they do not provide numbers correlated

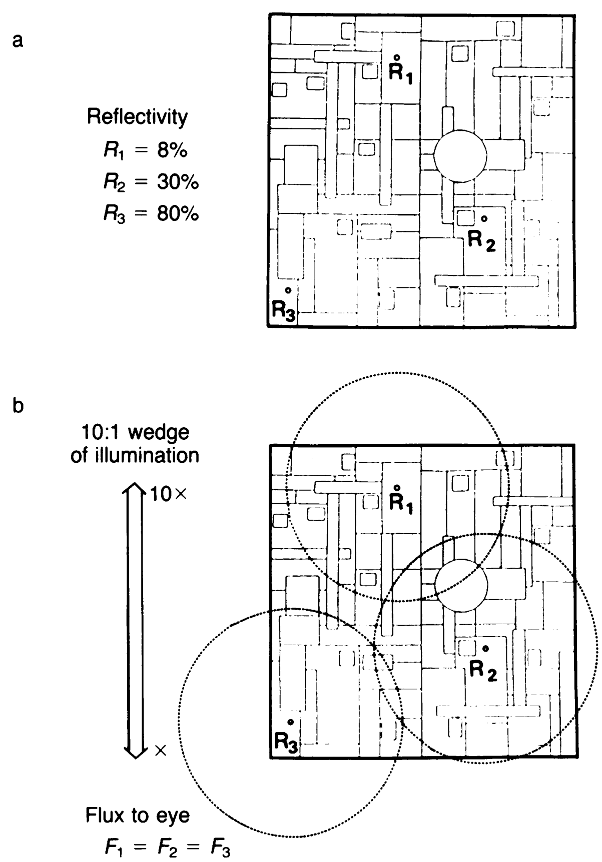

FIG. 1. A collage of black, white, and grey matte papers is shown in *a*. R_1 reflects 8% of the light falling on it, R_2 reflects 30%, and R_3 reflects 80%. In *b*, when the illumination is so adjusted by neutral wedges in the illuminator that the flux to the eye (F_1) from R_1 equals the flux to the eye (F_2) from R_2 equals the flux to the eye (F_3) from R_3, the observer will notice that the black stays black, the grey stays grey, and the white stays white, even though the three measured fluxes to the eye are identical. The circle in *b* represents a 16° diameter field.

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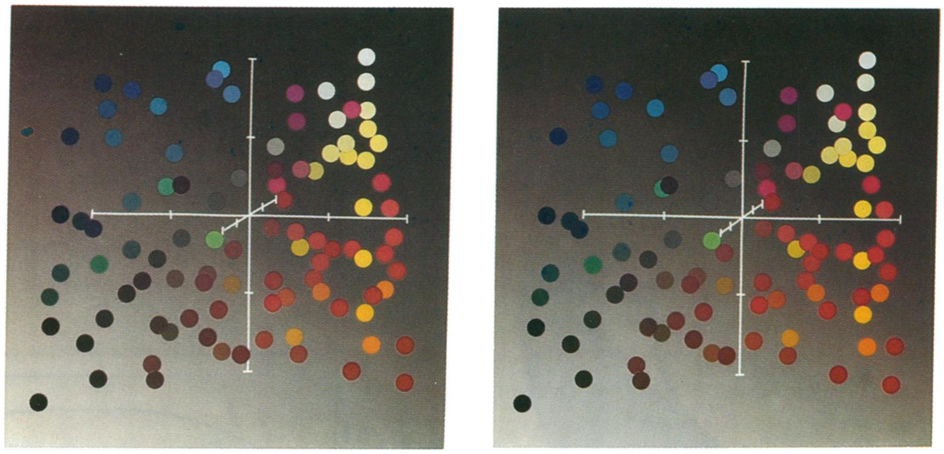

FIG. 2. The color three-space. Stereoscopic pair representing the left-eye view (*Left*) and the right-eye view (*Right*) of the color three-space populated, as indicated by designator theory, with dots of Color-Aid and Munsell papers. These photographs may be viewed in a stereoscope. The three designators for each paper were calculated according to an earlier algorithm (3). The results are consistent with those from the new technique described here.

with the fact that R_1 looks almost black, R_2 looks grey, and R_3 looks white. Clearly, the *overall* average cannot be used for the denominator in the relationship we seek.

If, instead of the overall average, we were to use the average flux from the contiguous areas (to give the value of the denominator) we would have to be concerned about the randomness of this kind of average because of the arbitrary reflectivities, and the smallness of the population, of contiguous areas.

A search for an operative compromise between an average taken over the whole field and an average taken over the contiguous areas gave promising results. For example, if, for the numerator, we use the flux per unit area over a 4-arc-min field, and for the denominator, we use the flux per unit area averaged over a 16° field (Fig. 1*b*), the ratios correlate with the appearance of the black, the grey, and the white areas, and the logarithms of these ratios give designators in the retinex color three-space (Fig. 2) at the correct locations for black, grey, and white. Even in this simple form, this measuring technique will satisfactorily locate a point in the color space in agreement with the three designators for each area in the colored Mondrian, and for the areas in the Macbeth color cards, for arrangements of fruits and vegetables and flowers, for clothing and real scenes.

Experiments, however, have shown that the flux from the extremes of the field of vision is somehow a significant contributor to the average. For example, with very large projected images, the radiation can be excluded from reaching the screen in an annular domain around the object of interest. Even though the radius of the outer circumference of the annulus may be relatively large, the average of the radiation beyond the annulus will, for each of the three wavebands, give a useful denominator; that is, it will produce a designator that meets our criteria of mapping lightnesses of equal value together in the color space.

Is there a pattern of sensitivity that would satisfy the conditions of the experiments we have described so far? A pattern that seems promising is one that, although suggested

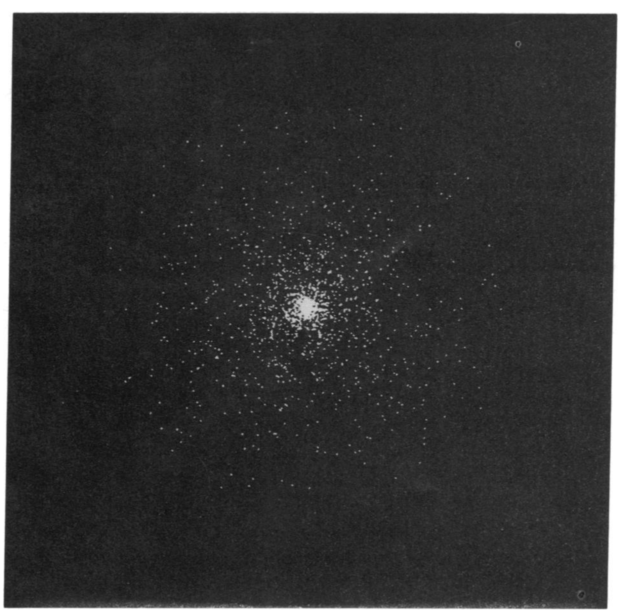

FIG. 3. Pattern of sensitivity for photometer reading of denominator. The angle from the center to the outermost dots (2.7 cm in this figure) is 12°. One-half the dots are inside a circle of radius 2°. (The corresponding radius of the numerator is 2 arc-min.) The density of dots, which was chosen empirically, here varies approximately as the inverse square of the radius.

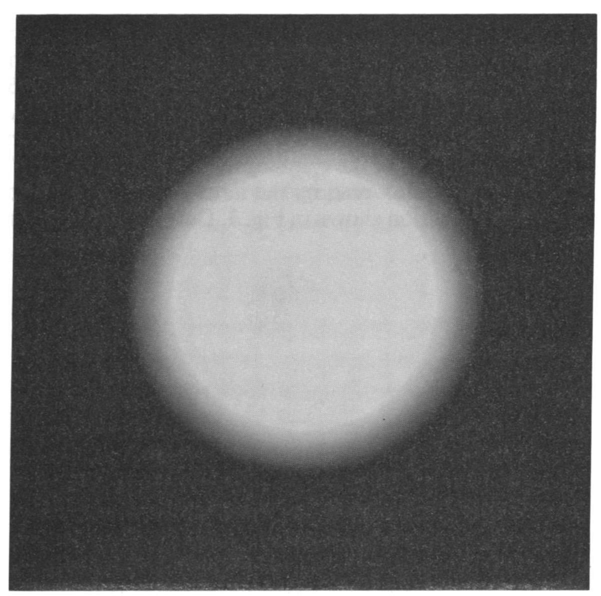

FIG. 4. Photograph of a white square on a black disc spinning. The Mach bands you see here do not exist for a photometer.

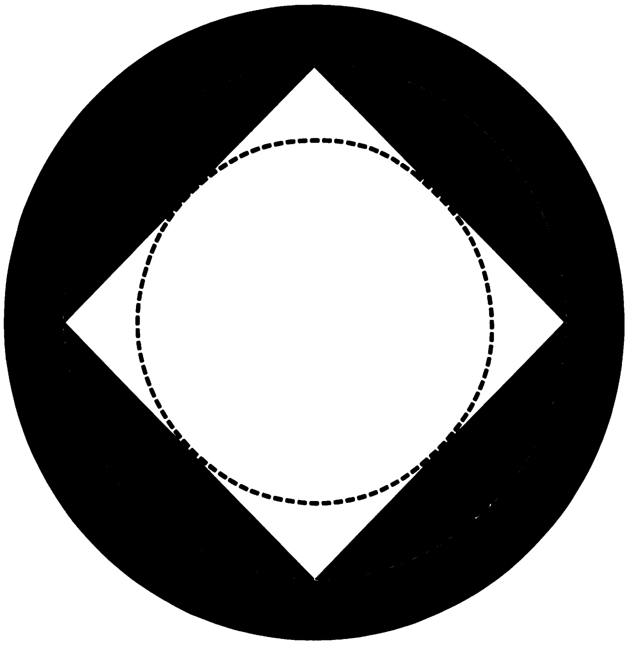

FIG. 5. Dotted lines show the location of Mach bands on a white square and black disc spinning.

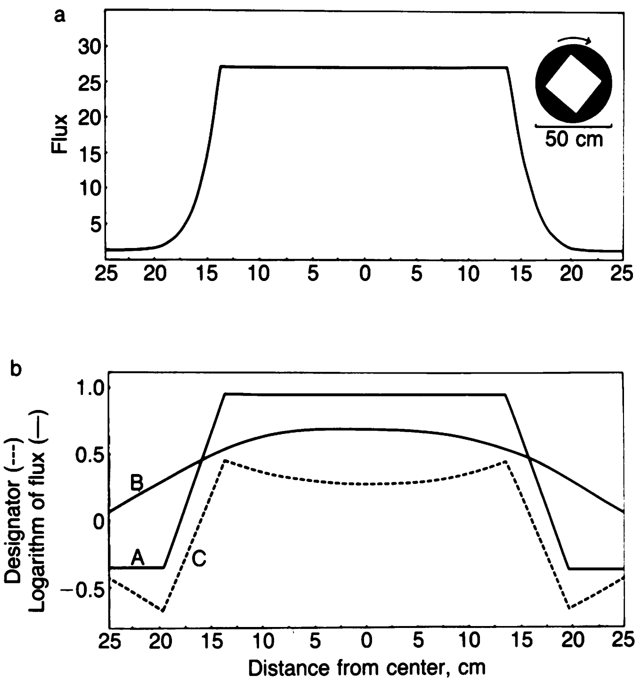

FIG. 6. Genesis of Mach bands. (*a*) A photometer trace made across a spinning white square on a black disc. The flux is shown on a linear scale. (*b*) Curve A is the logarithm of flux shown in *a*. Curve B is the logarithm of the flux measured along the same path with a large probe. Curve C, shown dotted, is the difference between curves A and B, namely the designator.

by the decreasing concentration of retinal cones with increasing radius, was actually arrived at empirically (Fig. 3). We have designed this pattern into the photometer so that the small angle reading for the numerator is taken through the center of it and the wide angle reading for the denominator utilizes the whole pattern, which now covers more than twice the area of the circle in Fig. 1*b*.

Fig. 4 is a photograph of a display made by spinning a white square on a black disc at high speed, whereupon the light band and the dark band appear in the positions shown in Fig. 5. These lines are commonly called Mach bands (6). A photometer that reads flux when scanned across the diameter of the spinning disc gives the track shown in Fig. 6*a* with no indication of the white line and dark line seen by the eye. The photometer implicitly required by retinex theory does not measure flux, but rather "lightness," the log of the ratio of the flux-per-unit-area from a very small central field to the average flux-per-unit-area over an extended field. It seemed extremely interesting, indeed, rather exciting, to ascertain whether traversing the spinning-square display with the "retinex photometer" would generate Mach bands. Fig. 6*b* shows the possibility that this is indeed the case. Solid line "A" is the logarithm of the flux read by the small-angle probe traversing the diameter of the spinning display. Solid line "B" is the logarithm of the flux read by the large-angle probe, which includes the distribution shown in Fig. 3. Dotted line "C" is the difference of the two logarithms, the quantity that would be the designator. The reader can see the angular dips in lightness on the left and on the right of the dotted line, which correspond to the black Mach band, and the peaks in lightness on the left and on the right of the dotted line, which correspond to the light Mach band. The depression at the center corresponds to the slightly dark area that one sees in the central region of a circular Mach band as shown in Fig. 4.

In summary, this alternative technique for computation of the designator in retinex theory has the appeal of intuitive simplicity, carries out all the tasks implicit in color constancy and has the competence not possessed by earlier algorithms for generating Mach bands.

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2. Land, E. H. (1959) *Proc. Natl. Acad. Sci. USA* **45,** 636–644.
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5. Land, E. H. (1986) *Vision Res.* **26,** 7–21.
6. Ratliffe, F. (1965) *Mach Bands: Quantitative Studies on Neural Networks in the Retina* (Holden Day, San Francisco).